Cambridge Assessment International Education
Cambridge International Advanced Level

FURTHER MATHEMATICS **9231/13**

Paper 1 **May/June 2018**

MARK SCHEME

Maximum Mark: 100

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2018 series for most Cambridge IGCSE™, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously 'correct' answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
SOI	Seen or implied
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

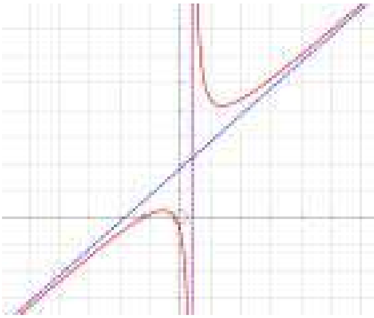
MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become 'follow through' marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1(i)	$\left(0+\dfrac{dy}{dx}\right)^3=(-1)^2+0\Rightarrow\dfrac{dy}{dx}=1$	**B1**	
1(ii)	$3\left(x+\dfrac{dy}{dx}\right)^2\left(1+\dfrac{d^2y}{dx^2}\right)$	**M1 A1**	Method mark for good attempt at implicit differentiation of LHS.
	$=2y\dfrac{dy}{dx}+1$	**B1**	Note: may expand bracket before differentiation but M1 is still for implicit differentiation
	$\Rightarrow 3\left(1+\dfrac{d^2y}{dx^2}\right)=-2+1\Rightarrow\dfrac{d^2y}{dx^2}=-\dfrac{4}{3}$	**A1**	
		5	

Question	Answer	Marks	Guidance
2(i)	$\dfrac{1}{ne^n}-\dfrac{1}{(n+1)e^{n+1}}=\dfrac{(n+1)e-n}{n(n+1)e^{n+1}}=\dfrac{n(e-1)+e}{n(n+1)e^{n+1}}$	**B1**	Verifies result (AG).
2(ii)	$S_N=\sum_{n=1}^{N}\left(\dfrac{1}{ne^n}-\dfrac{1}{(n+1)e^{n+1}}\right)=\left(\dfrac{1}{e}-\cancel{\dfrac{1}{2e^2}}+\cancel{\dfrac{1}{2e^2}}-\cancel{\dfrac{1}{3e^3}}+\right.$ ………… $\left.\cancel{\dfrac{1}{Ne^N}}-\dfrac{1}{(N+1)e^{N+1}}\right)$ SOI $=$	**M1**	Uses difference method to sum.
	$\dfrac{1}{e}-\dfrac{1}{(N+1)e^{N+1}}$.	**A1**	

Question	Answer	Marks	Guidance
2(iii)	S = $\frac{1}{e}$	**B1**	Finds S
	$(N+1)(S-S_N)$ $<10^{-3} \Rightarrow \frac{1}{e^{N+1}} < 10^{-3}$	**M1**	Attempts to find difference between sum and sum to infinity.
	$\Rightarrow e^{N+1} > 10^3$ $\Rightarrow$ least such N is 6.	**A1**	
		6	

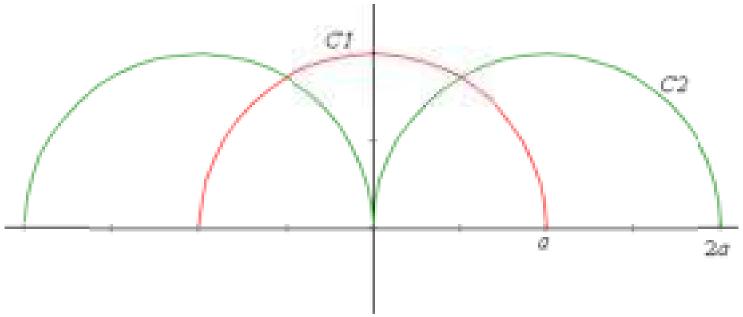
Question	Answer	Marks	Guidance
3(i)	$(c+is)^4 = c^4 + 4c^3(is) + 6c^2(-s^2) + 4c(is)^3 + (is)^4$	**M1**	Uses binomial theorem to expand $(c+is)^4$.
	$\Rightarrow \cos 4\theta = c^4 - 6c^2s^2 + s^4$	**M1 A1**	Takes real part, AG.
3(ii)	$\frac{\cos 4\theta}{\cos^4\theta} = \tan^4\theta - 6\tan^2\theta + 1$	**M1 A1**	Divides through by $\cos^4\theta$
	Let $x = \tan\theta, then\, x^4 - 6x^2 + 1 = 0 \Rightarrow \cos 4\theta = 0$ $\Rightarrow 4\theta = \pm\frac{\pi}{2} + 2m\pi,\ m \in \mathbb{Z}$	**dM1**	Following on from finding correct quartic Solves $\cos 4\theta = 0$.
	Roots are $\tan q\pi$ where $q = \frac{1}{8}, \frac{3}{8}, \frac{5}{8}, \frac{7}{8}$. (q > 0)	**A1 A1**	**Alt methods:** Solves tan(4θ) = (4t – t^3) / (t^4 – 6t^2 + 1) and tan(4θ) = ∞. Solves equation in cot(θ) after dividing by $\sin^4\theta$
		8	

Question	Answer	Marks	Guidance
4(i)	$(-6,0),(-1,0)$	**B1**	States points of intersection with x-axis.
	$(0,-3)$	**B1**	States y-intercept
4(ii)	One asymptote is $x=2$.	**B1**	
	$y=x+9+\dfrac{24}{x-2} \Rightarrow$ other asymptote is $y=x+9$.	**M1 A1**	By inspection or long division. A0 if error in division
4(iii)		**B1**	Sketches axes and asymptotes, labelled or to scale
		B1	Upper branch correctly located and orientated.
		B1	Lower branch correctly located and orientated. Penalise at most 1 mark for poor forms at infinity
		8	

Question	Answer	Marks	Guidance
5(i)	$\mathbf{Ae} = \lambda\mathbf{e}$ **SOI**	**B1**	
	$\mathbf{A}^3\mathbf{e} = \mathbf{A}^2(\mathbf{Ae}) = \lambda\mathbf{A}(\mathbf{Ae}) = \lambda^2(\mathbf{Ae}) = \lambda^3\mathbf{e}$	**M1**	Substitutes for $\mathbf{Ae}$
	So eigenvalue is λ^3. Special case: states eigenvalue is λ^3 B1	**A1**	
5(ii)	Eigenvalues of $\mathbf{A}$ are 2 and 3.	**B1**	
	Eigenvectors of $\mathbf{A}$ are $\begin{pmatrix}1\\1\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$.	**M1**	AEF (allow any non-zero scalar multiple).
	So $\mathbf{P} = \begin{pmatrix}1 & 0\\1 & 1\end{pmatrix}$	**A1**	**Alt method**: Find $\mathbf{A}^3 + \mathbf{I} = \begin{pmatrix}9 & 0\\-19 & 28\end{pmatrix}$ B1
	and $\mathbf{D} = \begin{pmatrix}2^3+1 & 0\\0 & 3^3+1\end{pmatrix} = \begin{pmatrix}9 & 0\\0 & 28\end{pmatrix}$ Columns of **P** and **D** can be permuted, but must match.	**M1** **A1FT**	Eigenvalues (9, 28) and vectors $\begin{pmatrix}1\\1\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$ M1, A1. **P** and **D** FT on eigenvalues M1, A1FT
		8	

Question	Answer	Marks	Guidance
6(i)	Substitutes $x=\frac{y+1}{3}$	M1	Accept substitution of $y=3x-1$ into given equation and derivation of equation in x.
	Obtains the given result	A1	AG.
6(ii)	$S_3=2S_1+7\times 3$	M1	Uses $y^3=2y+7$. Or uses formula for Σ $(3\alpha-1)^3$
	$=21$	A1	
6(iii)	$S_{-1}=\frac{(3\alpha-1)(3\beta-1)+(3\alpha-1)(3\gamma-1)+(3\beta-1)(3\gamma-1)}{(3\alpha-1)(3\beta-1)(3\gamma-1)}=\frac{-2}{7}$.	M1 A1	Award M1A1 if $S_{-1}=-\frac{2}{7}$ written down directly.
	$7S_{-2}=S_1-2S_{-1}$	M1	Uses $7y^{-2}=y-2y^{-1}$.
	$s_{-2}=\frac{4}{49}$	A1	
	Alt method: $S_{-2}=\sum\frac{1}{(3\alpha-1)^2}=\frac{\sum(3\alpha-1)^2(3\beta-1)^2}{(3\alpha-1)^2(3\beta-1)^2(3\gamma-1)^2}=$	M1 A1	**Alt method**: Finds cubic with roots $\frac{1}{3\alpha-1}$, etc. M1 $7z^3+2z^2-1=0$ A1 Uses $S_2=(S_1)^2-2x\Sigma\alpha\beta$ M1 $=\frac{4}{49}$ A1
	$\frac{(\sum(3\alpha-1)(3\beta-1))^2-2(3\alpha-1)(3\beta-1)(3\gamma-1)(\sum(3\alpha-1))}{(3\alpha-1)^2(3\beta-1)^2(3\gamma-1)^2}$	M1	
	$=\frac{(-2)2-2(7)(0)}{7^2}=\frac{4}{49}$	A1	
		8	

Question	Answer	Marks	Guidance
7(i)	Solve two equations $9+2\lambda=7+2\mu$ $13+3\lambda=-2-3\mu$	M1	
	to obtain $\lambda=-3$ and $\mu=-2$.	A1	
	Use third equation to obtain $a=2$.	A1	
7(ii)	Normal to the plane is $\mathbf{n}=-12\mathbf{i}+4\mathbf{k}$	M1 A1	
	Perpendicular distance $=\frac{1}{\sqrt{160}}\begin{pmatrix}-6\\6\\-8\end{pmatrix}\cdot\begin{pmatrix}-12\\0\\4\end{pmatrix}=\sqrt{10}=3.16$ **Alt method:** Finds equation of plane M1, A1: Finds foot of perpendicular from P to plane M1 Hence length A1 **Alt method:** Find equation of plane 3x – z + 7 = 0 M1, A1 Use formula $\frac{\lvert 3\times3+0\times1-6+7\rvert}{\sqrt{9+1}}=\sqrt{10}$ M1, A1	M1 A1	
7(iii)	Cross product of direction of $\boldsymbol{P}$ to l_2 with direction of l_2 $\begin{pmatrix}-6\\6\\-8\end{pmatrix}\times\begin{pmatrix}-1\\2\\-3\end{pmatrix}=\begin{pmatrix}-2\\-10\\-6\end{pmatrix}$	M1 A1	**Alt method:** Find N (foot of perpendicular) in terms of parameter and uses scalar product with n to find parameter M1, A1 so PN = $\begin{pmatrix}3\\0\\-1\end{pmatrix}$
	Perpendicular distance from $\boldsymbol{P}$ to l_2 is $\frac{\lvert 2\mathbf{i}+10\mathbf{j}+6\mathbf{k}\rvert}{\lvert -\mathbf{i}+2\mathbf{j}-3\mathbf{k}\rvert}=\sqrt{10}=3.16$	M1 A1	Find length PN M1, A1
		11	

Question	Answer	Marks	Guidance
8(i)	$a = 2a\lvert\cos\theta\rvert \Rightarrow \cos\theta = \pm\frac{1}{2}$	M1	Eliminates r .
	$\left(a, \frac{\pi}{3}\right)$ and $\left(a, \frac{2\pi}{3}\right)$	A1	Both points needed for A1.
8(ii)	 	B1	Semicircle for C1 including r = a.
		B1	Half of C2 including r = 2a.
		B1	Other half of C2 and line of symmetry.

Question	Answer	Marks	Guidance
8(iii)	$4a^2\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}\cos^2\theta\,\mathrm{d}\theta$	**M1**	Finds area of segment OP_1 and OP_2 of C_2
	$=2a^2\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}\cos 2\theta+1\,\mathrm{d}\theta$	**M1**	Uses $\cos^2\theta=\frac{1}{2}(\cos 2\theta+1)$
	$=2a^2\left[\frac{1}{2}\sin 2\theta+\theta\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}= \ 2a^2\left(\frac{\pi}{2}-\left(\frac{\sqrt{3}}{4}+\frac{\pi}{3}\right)\right)= \ a^2\left(\frac{\pi}{3}-\frac{\sqrt{3}}{2}\right)$	**A1**	Integrates correctly.
	Area $=\frac{\pi a^2}{6}-a^2\left(\frac{\pi}{3}-\frac{\sqrt{3}}{2}\right)=-\frac{\pi a^2}{6}+\frac{a^2\sqrt{3}}{2}$	**M1** **A1FT**	M1 for subtracting ‘their’ OP_1P_2 from $\frac{\pi a^2}{6}$
		10	

Question	Answer	Marks	Guidance
9(i)	$P_n: \quad u_n = 4\left(\frac{5}{4}\right)^n + 3$ Let $n=1$ then $4\left(\frac{5}{4}\right)+3=8 \Rightarrow P_1$ true.	B1	States proposition. Proves base case.
	Assume P_k is true for some k. Then	B1	States inductive hypothesis.
	$u_{k+1} = \frac{1}{4}\left(5\left(4\left(\frac{5}{4}\right)^k + 3\right) - 3\right) = \textit{correct step}$	M1	Proves inductive step.
	$= 4\left(\frac{5}{4}\right)^{k+1} + 3,$	A1	
	So $P_k \Rightarrow P_{k+1}$. Therefore, by induction, P_n is true for all positive integers.	A1	States conclusion.
9(ii)	$(u_n - 3)x^n = 4x^n\left(\frac{5}{4}\right)^n = 4\left(\frac{5x}{4}\right)^n$ so r = $\left(\frac{5x}{4}\right)$	M1	
	So series is convergent for $-1 < \frac{5x}{4} < 1 \Rightarrow -\frac{4}{5} < x < \frac{4}{5}$	A1	

Question	Answer	Marks	Guidance
9(iii)	$\sum_{n=1}^{N}\ln(u_n-3)=\sum_{n=1}^{N}\ln\left(4\left(\frac{5}{4}\right)^n\right)$ $=\left(\ln\frac{5}{4}\right)\sum_{n=1}^{N}n+\sum_{n=1}^{N}\ln 4$	M1	Alt method: $\sum_{n=1}^{N}\ln(u_n-3)=ln\prod 4\left(\frac{5}{4}\right)^n$ M1 $= \ln 4^{N}\prod_{1}^{N}\left(\frac{5}{4}\right)^n$ $= N\ln4+ \ln\left(\frac{5}{4}\right)^{\Sigma n}$
	$=\frac{1}{2}N(N+1)\ln\frac{5}{4}+N\ln 4$ Use $\sum_{n=1}^{N}n=\frac{1}{2}N(N+1)$.	M1	$= N\ln4 + \frac{N(N+1)}{2}\ln(\frac{5}{4})$ M1
	$=N^2\ln\frac{\sqrt{5}}{2}+N\ln(2\sqrt{5})\Rightarrow a=\frac{\sqrt{5}}{2},\ b=2\sqrt{5}$ oe Alt method: Writes series as an AP M1, uses summation formula M1 Correct answer A1	A1	$=N^2\ln\frac{\sqrt{5}}{2}+N\ln(2\sqrt{5})\Rightarrow a=\frac{\sqrt{5}}{2},\ b=2\sqrt{5}$ A1
		10	

Question	Answer	Marks	Guidance
10(i)	$y'=x+x't$	B1	
	$y''=x'+x'+tx''$	B1	
	Substitute correctly	B1	AG
10(ii)	Auxiliary equation: $m^2+9=0\Rightarrow m=\pm3i$.	M1	Correct auxiliary
	CF $=A\cos 3t+B\sin 3t$.	A1	

Question	Answer	Marks	Guidance
	PI $y = \mathrm{a}t^2 + \mathrm{b}t + \mathrm{c}$ so $y' = 2\mathrm{a}t + \mathrm{b}$ and $y'' = 2\mathrm{a}$	**M1**	Differentiate twice and substitute
	$\mathrm{a} = \frac{1}{3}, \mathrm{b} = 0, \mathrm{c} = \frac{1}{27}$.	**A1**	
	$y = A\cos 3t + B\sin 3t + \frac{1}{3}t^2 + \frac{1}{27}$	**A1FT**	Their CF + their PI both in correct form
	$x = \frac{\pi}{9}$ when $t = \frac{\pi}{3}$ gives $A = \frac{1}{27}$.	**B1**	
	$y' = -3A\sin 3t + 3B\cos 3t + \frac{2}{3}t$	**M1**	Differentiating their y of equivalent difficulty
	$x' = \frac{2}{3}$ when $t = \frac{\pi}{3}$ gives $B = -\frac{\pi}{27}$.	**A1**	
	$x = \frac{\cos 3t - \pi\sin 3t + 9t^2 + 1}{27t}$	**A1**	AEF
		12	

Question	Answer	Marks	Guidance
11E(i)	$I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \mathrm{e}^x \cos x \,\mathrm{d}x=\left[\mathrm{e}^x \cos x\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}-\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \mathrm{e}^x(-)\sin x \,\mathrm{d}x$	**M1 A1**	The parts can be either way round.
	$=\left[\mathrm{e}^x \sin x\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}-I$	**M1**	Integrates by parts again.
	$\Rightarrow 2I=\mathrm{e}^{\frac{\pi}{2}}+\mathrm{e}^{-\frac{\pi}{2}}$ AG	**A1**	Alt method: Integrate Re($\mathrm{e}^{\mathrm{i}x}$)e^x by parts M1, M1, A1, A1

Question	Answer	Marks	Guidance
11E(ii)	$I_n = \left[\frac{e^{2x}}{2}\cos^n x\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} + \frac{n}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{2x}\cos^{n-1} x \sin x \, dx$	**M1**	Integrates by parts once.
	$= 0 - \frac{n}{4}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{2x}\left(\cos^n x - (n-1)\sin^2 x \cos^{n-2} x\right)dx$	**M1**	Integrates by parts again.
	$\Rightarrow 4I_n = n(n-1)\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{2x}\sin^2 x \cos^{n-2} x \, dx - nI_n$	**A1**	Simplifies to answer. (AG.)
	$(n+4)I_n = n(n-1)\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{2x}\left(1-\cos^2 x\right)\cos^{n-2} x \, dx$	**M1**	For splitting $\sin^2 x$
	$\Rightarrow (n+4)I_n = n(n-1)I_{n-2} - n(n-1)I_n$	**M1**	
	$\Rightarrow (n^2+4)I_n = n(n-1)I_{n-2}$	**A1**	AG

Question	Answer	Marks	Guidance
11E(iii)	$\left(\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} y^2\,\mathrm{d}x =\right) I_2 = \frac{1}{4}I_0 = \frac{1}{4}\left[\frac{\mathrm{e}^{2x}}{2}\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \frac{1}{8}\left(\mathrm{e}^{\pi} - \mathrm{e}^{-\pi}\right)$	**M1 A1**	Uses reduction formula to find I_2.
	$\bar{y} = \frac{I_2}{2I} = \frac{1}{8}\left(\frac{\mathrm{e}^{\pi} - \mathrm{e}^{-\pi}}{\mathrm{e}^{\frac{\pi}{2}} + \mathrm{e}^{-\frac{\pi}{2}}}\right)(= 0.575)$	**M1 A1**	Uses correct formula for $\bar{y}$.
		14	

Question	Answer	Marks	Guidance
11O(i)	$\begin{pmatrix} 1 & -2 & 0 & 0 \\ 2 & -5 & -3 & -2 \\ 0 & 5 & 15 & 10 \\ 2 & 6 & 18 & 8 \end{pmatrix} \rightarrow \cdots \rightarrow \begin{pmatrix} 1 & -2 & 0 & 0 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$	**M1 A1**	Reduces to echelon or reduced row echelon form. At least 2 manipulations…. Or finds 4 columns dependent, 3 columns independent
	$\dim V =$ rank $= 3$ $\quad . c_1 \quad c_2 \quad c_3 \quad c_4$	**A1**	
11O(ii)	$c_4 = c_3 - c_2 - 2c_1 \Rightarrow \mathbf{v}_4 = \mathbf{v}_3 - \mathbf{v}_2 - 2\mathbf{v}_1$	**M1 A1**	OE
11O(iii)	$\{\mathbf{v}_1, \mathbf{v}_2\ , \mathbf{v}_3\}$	**B1**	OE

Question	Answer	Marks	Guidance
11O(iv)	$\mathbf{v}_1 + \mathbf{v}_2 = \mathbf{M}\begin{pmatrix}1\\1\\0\\0\end{pmatrix}$ so particular solution is $\begin{pmatrix}1\\1\\0\\0\end{pmatrix}$	**M1 A1**	Finds particular solution.
	$\begin{aligned}x - 2y \qquad\qquad &= 0\\ y + 3z + 2t &= 0\\ z + t &= 0\end{aligned}$	**M1**	Finds basis for null space.
	So basis for null space is $\left\{\begin{pmatrix}2\\1\\-1\\1\end{pmatrix}\right\}$	**A1**	AEF
	$\mathbf{x} = \begin{pmatrix}1\\1\\0\\0\end{pmatrix} + \lambda\begin{pmatrix}2\\1\\-1\\1\end{pmatrix}$.	**M1 A1**	Finds general solution. Alt method: Set up equations M1,A1: Solve equations M1, A1: x = etc. M1, A1 or using augmented matrix.
11O(v)	Not closed under addition $\left(\begin{pmatrix}1\\0\\0\\0\end{pmatrix} + \begin{pmatrix}0\\1\\0\\0\end{pmatrix} = \begin{pmatrix}1\\1\\0\\0\end{pmatrix}\right)$	**M1**	Accept any valid reason with evidence.
	so not a vector space.	**A1**	
		14	